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Some fixed point generalizations are not real generalizations

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ABSTRACT

In this paper, we shall prove that some generalizations in fixed point theory are not real generalizations.

1. Introduction

It is known that common fixed point (and coincidence point) theorems are generalizations of fixed point theorems. Over the past few decades, there have been a lot of activity in fixed point theory and a number of authors took interest in generalizing fixed point theorems to coincidence point theorems and common fixed point theorems. In this paper, we shall show that some coincidence point and common fixed point generalizations in fixed point theory are not real generalizations as they could easily be obtained from the corresponding fixed point theorems. Here, we shall review only some results which appeared recently in the literature. This shows that authors should take care in obtaining real generalizations in fixed point theory.

2. Main results

In this section, we state and prove our main results. First, we give the following lemma which is a key result.

Lemma 2.1. *Let X be a nonempty set and $f : X \to X$ a function. Then there exists a subset $E \subseteq X$ such that $f(E) = f(X)$ and $f : E \to X$ is one-to-one.*

Proof. Define a multifunction $F : f(X) \to 2^X$ by $F(y) = \{x \in X : f(x) = y\}$. By using the axiom of choice, F has a selector, that is, there is a function $g : f(X) \to X$ such that $g(y) \in F(y)$ for all $y \in f(X)$. Note that, $f(g(y)) = y$ for all $y \in f(X)$. Now, put $E = \{g(y) : y \in f(X)\}$. It is clear that f is one-to-one on E and $f(E) = f(X)$. □

2.1. Contractive maps

Let (E, τ) be a topological vector space and P a subset of E. Then, P is called a cone whenever

(i) P is closed, nonempty and $P \neq \{0\}$,

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