

# Approximating common solution of variational inequality problems for two monotone mappings in Banach spaces

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**Abstract** In this paper, we introduce an iterative process which converges strongly to a common solution of variational inequality problems for two monotone mappings in Banach spaces. Furthermore, our convergence theorem is applied to the convex minimization problem. Our theorems extend and unify most of the results that have been proved for the class of monotone mappings.

**Keywords** Generalized projection ·  $\gamma$ -Inverse strongly monotone mappings · Monotone mappings · Strong convergence · Variational inequality problems

## 1 Introduction

Let  $E$  be a real Banach space with dual  $E^*$ . We denote by  $J$  the normalized duality mapping from  $E$  to  $2^{E^*}$  defined by

$$Jx := \{f^* \in E^* : \langle x, f^* \rangle = \|x\|^2 = \|f^*\|^2\},$$

where  $\langle \cdot, \cdot \rangle$  denotes the duality pairing. It is well known that if  $E$  is smooth then  $J$  is single-valued and demi-continuous, and if  $E$  is uniformly smooth, then  $J$  is uniformly continuous on bounded subsets of  $E$ . Moreover, if  $E$  is a reflexive and strictly convex Banach space with a strictly convex dual, then  $J^{-1}$  is single valued, one-to-one,

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