Computers and Mathematics with Applications 62 (2011) 1200–1214

Contents lists available at ScienceDirect

Computers and Mathematics with Applications

journal homepage: www.elsevier.com/locate/camwa

Existence theory for anti-periodic boundary value problems of fractional differential equations and inclusions

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ARTICLE INFO

Keywords:
Fractional differential equations
Inclusions
Anti-periodic boundary conditions
Existence
Multivalued maps
Fixed point theorems

ABSTRACT

This paper studies the existence of solutions for nonlinear fractional differential equations and inclusions of order $q \in (3, 4]$ with anti-periodic boundary conditions. In the case of inclusion problem, the existence results are established for convex as well as nonconvex multivalued maps. Our results are based on some fixed point theorems, Leray–Schauder degree theory, and nonlinear alternative of Leray–Schauder type. Some illustrative examples are discussed.

1. Introduction

The subject of fractional calculus has recently gained much momentum and a variety of problems involving differential equations and inclusions of fractional order have been addressed by several researchers. Fractional differential equations appear naturally in a number of fields such as physics, polymer rheology, regular variation in thermodynamics, biophysics, blood flow phenomena, aerodynamics, electro-dynamics of complex medium, viscoelasticity, Bode analysis of feedback amplifiers, capacitor theory, electrical circuits, electro-analytical chemistry, biology, control theory, fitting of experimental data, etc. [1–4]. For some recent work on fractional differential equations and inclusions, see [5–13] and the references therein.

Anti-periodic boundary value problems occur in the mathematical modeling of a variety of physical processes and have recently received considerable attention. For examples and details of anti-periodic boundary conditions, see [14–18] and the references therein.

In this paper, we discuss some existence results for anti-periodic boundary value problems of differential equations and inclusions of fractional order $q \in (3, 4]$. Precisely, we consider the following problems:

$$\begin{cases} {}^cD^q x(t) = f(t, x(t)), \quad t \in [0, T],\ T > 0,\ 3 < q \le 4, \\ x(0) = -x(T), \qquad x'(0) = -x'(T), \qquad x''(0) = -x''(T), \qquad x'''(0) = -x'''(T), \end{cases} \tag{1.1}$$

where ${}^cD^q$ denotes the Caputo fractional derivative of order q, f is a given continuous function, and

$$\begin{cases} {}^cD^q x(t) \in F(t, x(t)), \quad t \in [0, T],\ T > 0,\ 3 < q \le 4, \\ x(0) = -x(T), \qquad x'(0) = -x'(T), \qquad x''(0) = -x''(T), \qquad x'''(0) = -x'''(T). \end{cases} \tag{1.2}$$

In (1.2), $F : [0, T] \times \mathbb{R} \to \mathcal{P}(\mathbb{R})$ is a multivalued map, $\mathcal{P}(\mathbb{R})$ is the family of all subsets of $\mathbb{R}$.

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0898-1221/$ – see front matter
doi:10.1016/j.camwa.2011.03.001