

## MONOTONE ITERATION SCHEME FOR IMPULSIVE THREE-POINT NONLINEAR BOUNDARY VALUE PROBLEMS WITH QUADRATIC CONVERGENCE

BASHIR AHMAD, AHMED ALSAEDI, AND DOA'A GAROUT

**ABSTRACT.** In this paper, we consider an impulsive nonlinear second order ordinary differential equation with nonlinear three-point boundary conditions and develop a monotone iteration scheme by relaxing the convexity assumption on the function involved in the differential equation and the concavity assumption on nonlinearities in the boundary conditions. In fact, we obtain monotone sequences of iterates (approximate solutions) converging quadratically to the unique solution of the impulsive three-point boundary value problem.

### 1. Introduction

In recent years, a number of research papers have dealt with dynamical systems with impulse effect as a class of general hybrid systems. Impulsive hybrid systems are composed of some continuous variable dynamic systems along with certain reset maps that define impulsive switching among them. It is the switching that resets the modes and changes the continuous state of the system. There are three classes of impulsive hybrid systems, namely, impulsive differential systems [33, 41], sampled data or digital control system [30, 44] and impulsive switched system [19, 24]. Applications of such systems include air traffic management [43], automotive control [5, 9], real-time software verification [6], transportation systems [37, 45], manufacturing [39], mobile robotics [10], process industry [25], etc. In fact, hybrid systems have a central role in embedded control systems that interact with the physical world. Using hybrid models, one may represent time and event-based behaviors more accurately so as to meet challenging design requirements in the design of control systems for problems such as cut-off control and idle speed control of the engine. For more details, see [7] and the references therein.

Impulsive differential equations, which provide a natural description of observed evolution processes, are regarded as important mathematical tools for

---

Received December 14, 2006.

2000 *Mathematics Subject Classification.* 34A37, 34B10, 34B15.

*Key words and phrases.* quasilinearization, three-point boundary value problems, quadratic convergence.